

Topologically-protected quantum memory interfacing atomic and superconducting qubits

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We propose a scheme to manipulate a kind of topological spin qubits realized with cold atoms in a one-dimensional optical lattice. In particular, by introducing a quantum optomechanical setup, we are able to first transfer a superconducting qubit state to an atomic state and then to store it into the topological spin qubit. In this way, an efficient topological quantum memory may be constructed for the superconducting qubit. Therefore, we can consolidate the advantages of both the noise resistance of topological qubits and the scalability of superconducting qubits in this hybrid architecture.

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Quantum computation has attracted much attention as they are generally believed to be capable of solving diverse classes of hard problems. Superconducting circuit is one of the most promising candidates serving as its hardware implementation as they are potential scalable [1]. As superconducting qubits are quite sensitive to the external environments and background noises, their decoherence time is generally rather short [1]. Therefore, how to fight against this short decoherence is one of the big challenges in this solid state implementation of quantum computation. A promising strategy out of this difficulty is based on the topological idea [2]: a topological qubit is largely insensitive to major sources of local noises for longer times, and thus constituting an efficient topological quantum memory (TQM).

Recently, with the potential applications in topological quantum computation, topological matters have attracted renewed interests [3, 4]. In particularly, time-reversal invariant topological insulators [5–9] have been reported experimentally, and thus greatly stimulates the study of topological phases [4]. In engineering topological phases, a spin-orbit (SO) interaction usually plays an important role. Therefore, with recent great achievements in realizing artificial SO interaction in cold atom system [10–15], it becomes a new platform to probe topological phases in a fully controllable way. Recently, Liu *et al.* [16] proposed to observe and manipulate topological edge spins in one-dimensional (1D) optical lattice with experimentally realized SO interaction, where large chains of atoms are directly possible [17]. The nontrivial topology supports two degenerate zero modes, which are topological stable, and thus can be used to define a topological spin qubit (TSQ).

In this paper, we propose a scheme to realize an interface between this TSQ and a solid-state superconducting qubit.

This hybrid system may allow us to combine the advantages of both the noise resistance of topological qubit and the scalability of superconducting qubit. With the help of cavity assisted interaction [18], we show that local operations can be achieved for the TSQ. Our particular interest lies in using this TSQ as a TQM, where we can store quantum information of atoms as well as superconducting qubits. To store a quantum state of a superconducting qubit into a TSQ formed with the atomic lattice, as shown in Fig. 1(a), we firstly transfer the superconducting qubit state to an ancillary atomic qubit [pink circle in Fig. 1(a)] and then store the ancillary atomic qubit state into the TSQ. As the superconducting and ancillary atomic qubits are of vastly different frequencies, a quantum optomechanical scenario [19] is needed, where a mechanical oscillator mediates the coherent coupling of the microwave and optical photons [20, 21]. The superconducting qubit [blue rectangle in Fig. 1(a)] interacts with a microwave mode in a circuit QED scenario [22], while the atoms interact with the optical cavity mode. When switching on the intra-cavity interaction, one can obtain high fidelity state transfer from a superconducting qubit to the ancillary atomic qubit. By ignoring the coupling to the microwave photon, which can be achieved by switching off the intra-cavity interaction, we show that a TQM for the ancillary atom can be constructed. Combining the two processes become particularly arresting as we can store a superconducting qubit state into the TQM in this hybrid architecture.

We begin by a brief review of the model of Liu *et al.* [16], which is based on quasi-1D cold fermions with three-level Λ configuration trapped in an optical lattice. As shown in Fig. 1(b), the transitions $|0\rangle \rightarrow |e\rangle$ and $|1\rangle \rightarrow |e\rangle$ are induced by the laser fields with Rabi-frequency $\Omega_1 = \Omega \sin(k_0 x)$ and $\Omega_2 = \Omega \cos(k_0 x)$, respectively. Meanwhile, the two coupling lasers have a large one-photon detuning $|\Delta| \gg \Omega$ and a small two-photon detuning $|\delta_0| \ll \Omega$. The effect of the small two-photon detuning is equivalent to a Zeeman field along z axis $\Gamma_z = \hbar\delta_0/2$, which can be precisely controlled. Adiabatically eliminating the excited state $|e\rangle$ yields the following effective

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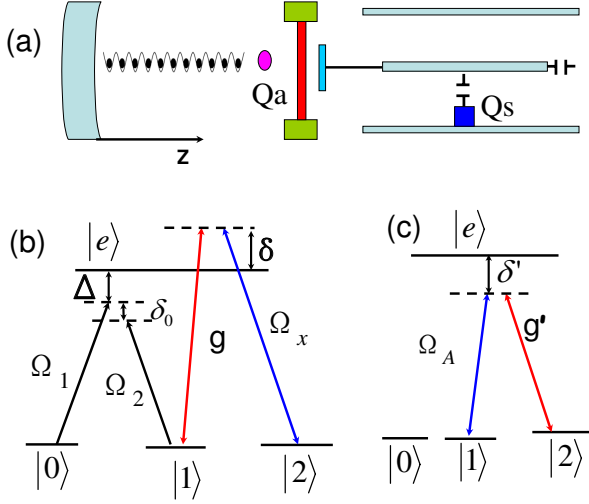


FIG. 1: (a) The hybrid architecture consists with two-cavity optomechanical system. A mechanical oscillator (red) mediates the coupling of the optical (left) and microwave photons (right). A 1D optical lattice, constructing our TQM, is inside an optical cavity. An ancillary atom (pink circle) is incorporated to engineer the cavity photon state and its information can be stored in the TQM. (b) The level structure of atoms in the optical lattice. The transitions $|0\rangle \rightarrow |e\rangle$ and $|1\rangle \rightarrow |e\rangle$ are induced by the lasers in a large one-photon detuning $|\Delta| \gg \Omega$ and a small two-photon detuning $|\delta_0| \ll \Omega$. The transition $|1\rangle \rightarrow |e\rangle$ is dispersively coupled to the cavity field to achieve the QND Hamiltonian. To obtain effective switch of the cavity-assisted interaction, a strong control laser of Rabi frequency Ω_x driving the transition of $|2\rangle \rightarrow |e\rangle$ is also introduced. (c) The level structure of the ancillary atom.

Hamiltonian

$$H_{\text{eff}} = \frac{p_x^2}{2m} + \sum_{\sigma=0,1} [V_\sigma(x) + \Gamma_z \sigma_z] |\sigma\rangle \langle \sigma| - [M(x)|0\rangle \langle 1| + \text{H.c.}], \quad (1)$$

where $M(x) = M_0 \sin(2k_0 x)$ with $M_0 = \frac{\hbar \Omega^2}{2\Delta}$ representing an additional laser-induced Zeeman field along x axis. To form a 1D lattice, the optical dipole trapping potentials are chosen as $V_0(x) = V_1(x) = -V_0 \cos^2(2k_0 x)$ with the lattice trapping frequency $\omega = (8V_0 k_0^2/m)^{1/2}$ [17]. The states $|0\rangle$ and $|1\rangle$ are defined as spin up and down for a pseudo-spin, respectively. Redefining the spin-down operator $\hat{c}_{j\downarrow} \rightarrow e^{i\pi x_j/a} \hat{c}_{j\downarrow}$ with a being lattice constant, the tight-binding Hamiltonian reads [16]

$$H = -t_s \sum_{\langle i,j \rangle} (\hat{c}_{i\uparrow}^\dagger \hat{c}_{j\uparrow} - \hat{c}_{i\downarrow}^\dagger \hat{c}_{j\downarrow}) + \sum_i \Gamma_z (\hat{n}_{i\uparrow} - \hat{n}_{i\downarrow}) + [\sum_j t_{\text{so}}^{(0)} (\hat{c}_{j\uparrow}^\dagger \hat{c}_{j+1\downarrow} - \hat{c}_{j\uparrow}^\dagger \hat{c}_{j-1\downarrow}) + \text{H.c.}], \quad (2)$$

where $t_s = \int dx \phi_{s\sigma}^{(j)}(x) [\frac{p_x^2}{2m} + V] \phi_{s\sigma}^{(j+1)}(x)$, $\hat{n}_{i\sigma} = \hat{c}_{i\sigma}^\dagger \hat{c}_{i\sigma}$, and $t_{\text{so}}^{(0)} = \frac{\Omega^2}{\Delta} \int dx \phi_s(x) \sin(2k_0 x) \phi_s(x-a)$. In the k -space $H_k = -\sum_{k,\sigma,\sigma'} \hat{c}_{k,\sigma}^\dagger [d_z(k)\sigma_z + d_y(k)\sigma_y]_{\sigma,\sigma'} \hat{c}_{k,\sigma'}$ with

$d_y = 2t_{\text{so}}^{(0)} \sin(ka)$ and $d_z = -\Gamma_z + 2t_s \cos(ka)$. This Hamiltonian describes a nontrivial topological insulator for $|\Gamma_z| < 2t_s$ and otherwise a trivial insulator, with a bulk gap $E_g = \min\{|2t_s - |\Gamma_z||, 2|t_{\text{so}}^{(0)}|\}$. The nontrivial topology supports two degenerate boundary modes, and each edge state equals one-half of a single spin, similar to the relation between a Majorana fermion and a complex fermion in topological superconductors [23]. As a result, the zero modes are robust to local noises, and thus leads to a TSQ which we use as a TQM. Local operations upon the TSQ can be achieved by applying a local Zeeman term $\mathbf{B}_y = \Gamma_0 \sigma_y$ or $\mathbf{B}_z = \Gamma_0 \sigma_z$ [16]. When $|\Gamma_0| > 2|t_{\text{so}}^{(0)}|$ a mass domain is created, associated with two midgap spin states $|\psi_\pm\rangle$ localized around the two edges at $x = x_{1,2}$ of the defined TSQ. Let $|\psi_+\rangle$ be initially occupied, reducing $|\Gamma_0|$ smoothly can open the coupling in $|\psi_\pm\rangle$ and lead TSQ state to evolve. If we apply the local Zeeman field along z (y) axis, we obtain the TSQ to evolve in the x - y (x - z) plane [16].

We then show how to store the state of an auxiliary atomic state into the TSQ. For our TQM purpose, we need controlled manipulation of the TSQ, and thus a controlled local Zeeman field. To achieve this, we propose the implementation with cavity-assisted quantum nondemolition (QND) hamiltonian [18]. Therefore, we introduce another level, and thus the fermions are in four-level Tripod configuration. For ^6Li and ^{40}K atoms, we can find appropriate hyperfine levels to meet our need. The coupling structure is shown in Fig. 1(b). To implement a QND Hamiltonian $H_{\text{QND}} = \chi a^\dagger a \sum_l \sigma_l^z$, we set the cavity mode couples the transition of $|1\rangle \leftrightarrow |e\rangle$ with a strength g and blue detuning δ , where $\chi = g^2/(2\delta)$ and $a(a^\dagger)$ is the annihilation (creation) operator for the cavity photon. Note that TSQ can also be obtained in the middle area in the lattice, e.g., $x \in [x_1, x_2]$, by creating mass domain. As the cavity assisted interaction is of the always-on QND nature, if we want the interaction is only induced among atoms within certain area, we should be able to decouple this coupling. This can be done by another laser that couples the transition of excited state to another level with a Rabi frequency Ω_x , i.e., $|2\rangle \leftrightarrow |e\rangle$, and configures a two-photon resonance with the QND coupling, as shown in the Fig. 1(b). When $\Omega_x \gg g$, the destructive interference of excitation pathways from the two transition ensures that the so-called dark state, which decoupled the atoms from interacting with both optical fields [24]. This QND Hamiltonian preserves the photon number n_c of the cavity mode. Within the $n_c \in \{0, 1\}$ subspace, the evolution of the QND Hamiltonian during the interaction time $\tau = \pi/(2\chi)$ yields [18]

$$U = \exp[-i\tau H_{\text{QND}}] = \begin{cases} \mathbf{I} & \text{for } n_c = 0 \\ (-i)^N \prod_l \sigma_l^z & \text{for } n_c = 1 \end{cases} \quad (3)$$

where N is the number of the selected atoms. If the cavity is initially prepared in the $n_c = 1$ state, the global operation reduces to the string operation $U_z = \prod_l \sigma_l^z$. Given the fact that all string operators are equivalent to U_z up to local single spin rotations [18]: $U_x = \prod_l \sigma_l^x = H U_z H$ and $U_y = \prod_l \sigma_l^y = R U_z R$, where $H = \prod_l H_l$ and $R = \prod_l R_l$ with $H_l = (\sigma_l^x + \sigma_l^z)/\sqrt{2}$ being the Hadamard rotation and

$R_l = \exp(-i\frac{\pi}{4}\sigma_l^z)$, since we can implement U_z , the single qubit operation on TSQ can then be implemented.

When the cavity state is in the superposition state of $\mu|0\rangle_c + \nu|1\rangle_c$, the global operation in Eq. (3) reduces to a controlled-string operation: $U_1 = |0\rangle_c\langle 0| \otimes I + |1\rangle_c\langle 1| \otimes U_z$. Here, we need to engineer the cavity number states. However, it is actually easier to control an ancillary atom rather than to directly manipulate the photon number state. Therefore, we also put an ancillary atom in the optical cavity, which is used to achieve controlled-string operations between the ancilla atom and the TSQ. The level structure of the ancillary atom is shown in Fig. 1(c), the transitions of $|2\rangle \leftrightarrow |e\rangle$ and $|1\rangle \leftrightarrow |e\rangle$ are coupled by the cavity field with strength g' and a laser with Rabi frequency Ω_A , respectively. Meanwhile, the two couplings are in a two photon resonance with a red detuning δ' to the excited state $|e\rangle$. This coupling term is described by an effective Hamiltonian $H_e = \lambda(a^\dagger|2\rangle\langle 1| + a|1\rangle\langle 2|)$ with $\lambda = g\Omega_A/\delta'$. The starting point is that the cavity mode is in a vacuum state $|0\rangle$ and the ancilla atom is in an arbitrary superposition state of $\alpha|0\rangle_A + \beta|1\rangle_A$. The procedure is as follows. (1) An interaction of H_e for $t = \pi/\lambda$ coherently couple the cavity with the atom: $\alpha|0\rangle_A|0\rangle_c + \beta|2\rangle_A|1\rangle_c$. (2) The QND Hamiltonian for $\tau = \pi/(2\chi)$ on the above intermediate state is applied. (3) Applying H_e for another $t = \pi/\lambda$ will annihilate the cavity photon and restore the ancilla atom to its original state. In the steps (1) and (3), we have neglected a phase factor, which can be compensated by a sing-qubit rotation on the ancilla atom. In this way, one realizes a controlled-string operation conditioned on the state of the ancilla atom [18]

$$U_2 = |0\rangle_A\langle 0| \otimes I + |1\rangle_A\langle 1| \otimes U_z. \quad (4)$$

In particular, upon local single spin rotations, one can obtain the controlled operations condition on the ancillary atomic state $U_{cs}^z = |0\rangle_A\langle 0| \otimes I + |1\rangle_A\langle 1| \otimes S_{TSQ}^z$ and $U_{cs}^x = |0\rangle_A\langle 0| \otimes I + |1\rangle_A\langle 1| \otimes S_{TSQ}^x$ with $S_{TSQ}^{(x,z)}$ being the Pauli matrices for the TSQ by selecting appropriate atoms into the QND interaction.

With such controlled operations, one is able to access an efficient topological quantum memory [18]. For this purpose, we need a swap in gate that swaps the auxiliary atomic state $(\alpha|0\rangle + \beta|1\rangle)_A$ into the topological memory initialized in $|\psi_+\rangle$, which is $U_{in} = H_A U_{cs}^z H_A U_{cs}^x$ with H_A being the Hadamard rotation on the atom. For the inverse process, we need the swap out gate that swaps the stored information back to the auxiliary atom prepared in $|0\rangle_A$, which is $U_{out} = U_{cs}^x H_A U_{cs}^z H_A$. The swap in (out) process corresponds to write (read) process for our TQM.

We now turn to discuss the experimental feasibility of storing the auxiliary atomic state into the TQM. (1) The TSQ considered here is robust in the large N limit [23]. For finite N , the ground-state degeneracy is broken, and thus cause decoherence. However, the lifetime of the topological qubit is exponentially increased with the increasing of N . Therefore, for small N , one may already obtain a TSQ with very weak decoherence [16]. (2) To implement the the QND Hamiltonian, we use the large detuned scheme, which only requires large Purcell factor condition, i.e., $g^2 > \gamma\kappa$ with γ and κ

being the spontaneous decay of level $|e\rangle$ and the cavity decay rate, respectively. The effective spontaneous decay rate is suppressed to $\gamma_{eff} = \gamma g^2/\delta^2$. Therefore, as the selected atoms decay independently, the total probability for photon loss is $P_{loss}^N = (\kappa + N\gamma_{eff})\tau \geq 2\pi\sqrt{N/P} \approx 3\%$ with $P = g^2/(\kappa\gamma)$ being the Purcell factor for $N = 5$, $g/(2\pi) = 220$ MHz [25], $\gamma = 10$ MHz, and $\kappa = 1$ MHz. (3) Addressing error of the laser is associated with a finite spread around the lattice points of atoms, which results in a tiny coupling between the addressing beam and selected spins, and the finite lifetime for the state $|2\rangle$. The error probability associated with addressing each site is estimated to be $\varepsilon_0 = 1\%$ [24], which can also be further suppressed. Meanwhile, what we need is mainly global operations, which address no more than two sites for an operation. (4) The deviation of the QND interaction, which degrades the controlled string operation, can also be corrected by the quantum control techniques to arbitrarily high order [26]. (5) While for the controlled string operation, we further need the light-atom interface, i.e., the reversible state transfer between light and an atom. With a strong lase field $\Omega_A = 1$ GHz, the error rate can be obtained under $P_i = 1\%$. Finally, combining the above error channels, we may obtain a fidelity about 95% for a controlled operation U_{cs}^z . Both the read and write processes of the TQM need two controlled operations, a fidelity of $F_1 = 95\% \times 95\% \approx 90\%$ can be obtained for the storage of the auxiliary atomic state into the TQM.

We now show that by incorporating a hybrid system of quantum optomechanics [19], we can achieve the storage of a superconducting qubit state into our TQM. The combined setup is shown in Fig. 1(a), where we consider the case of two-cavity optomechanical system. For our quantum memory purpose, we can firstly transfer the states of superconducting qubit to the ancillary atomic qubit and then store the ancillary atomic qubit state into the TSQ as proposed previously. Therefore, we only need to consider the transfer the states of superconducting qubit to the ancillary atomic qubit in the following. In a circuit QED system, under the rotating-wave approximation, the interaction Hamiltonian takes Jaynes-Cummings form [22] $H_m = g_m(b^\dagger\sigma_s^- + b\sigma_s^+)$, where g_m is the coupling strength of the superconducting qubit to the microwave cavity, the subscript "s" stands for this operator belongs to a superconducting qubit, b and b^\dagger is the annihilation and creation operator of the cavity field, respectively. Similarly, an atomic qubit coupled to an optical cavity [25] is described by $H_o = g_o(a^\dagger\sigma_a^- + a\sigma_a^+)$, where the subscript "a" stands for atom. For our system, we consider that the optomechanical coupling is enhanced by strongly driving of each cavity, resulting in an effective linear couplings [27]. Assuming that each cavity is far into the resolved-sideband regime and is driven near the red-detuned mechanical sideband, in the interaction picture with respect to the two cavity drives, the interaction Hamiltonian reads [20, 27, 28]

$$H_c = G_1(da^\dagger + d^\dagger a) + G_2(db^\dagger + d^\dagger b), \quad (5)$$

where d is annihilation operator of the mechanical oscillator. The coupling between the mechanical resonator and cavity i is denoted as G_i , which can be controlled in time as

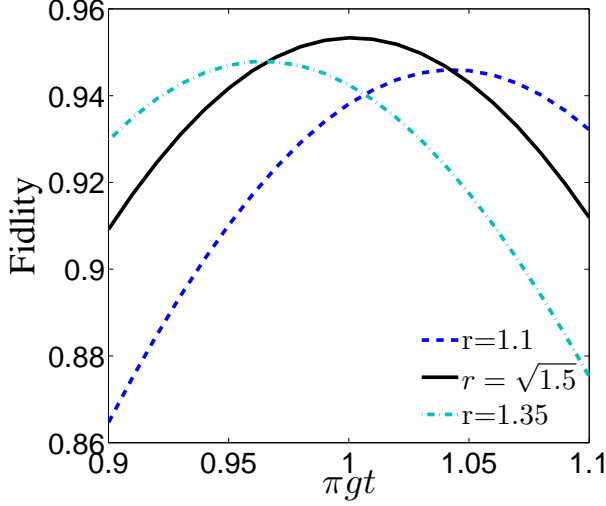


FIG. 2: The fidelity of the quantum state transfer between a superconducting qubit and the ancillary atom as a function of πgt . The parameters are $\kappa_d = 0.1$ MHz, $\kappa_b = \kappa_a = 1$ MHz, $\gamma_a = \gamma_s = 0.1$ MHz, $g = 6\pi$ MHz.

they are proportional to the external drive amplitude, and we choose $G_1 = G_2 = G$ here for simplicity. Then, the total Hamiltonian reads $H = H_o + H_m + H_c$, which conserves the total excitations and we restrict to the zero- and single-excitation subspaces. Transfer of the intra-cavity qubits states can be accomplished by modulating system parameters [29]. As the light-matter interaction is tunable, we can modulate $g_o = g_m = g$. The initial state of the atom and superconducting qubits are assumed as $|0\rangle_a$ and $|\psi_s\rangle = (\alpha|0\rangle_s + \beta|1\rangle_s)$, respectively. Deterministic quantum state transfer is obtained when $\exp(-itH)|000\rangle_c|0\rangle_a|\psi_s\rangle = |000\rangle_c|\psi_a\rangle|0\rangle_s$ is fulfilled, where $|\psi_a\rangle = \alpha|0\rangle_a + \beta|1\rangle_a$ and $|000\rangle_c$ means the three bosonic modes are all in the vacuum state.

Diagonalizing the total Hamiltonian in our considered subspace, after some algebra, we find that at time $t = \pi/g$ when the relation $2r^2 = (4k^2 - 1)$ is fulfilled, where $k = 1, 2, 3, \dots$ and $r = G/g$, deterministic state transfer is achieved when neglecting dissipation. To facilitate the transfer process, stronger coupling G [20] is preferable. Note that the adiabatic transfer protocols [28] for photonic states are not applicable for our purpose. To achieve adiabatic transfer of qubits state, $r \gg 1$ is required in order to single out only the dark bosonic mode, which results in much smaller g for a given G . Then, the time needed to complete the transfer will be much longer, and thus decoherence will cause considerable error. However, if very strong G is experimental accessible, e.g., $r = 20$, which means $G \sim 100$ MHz, we have numerically confirmed that one can eliminate the influence of mechanical mode decay on the state transfer process.

We next estimate the influence of dissipation to the state transfer process by integrating the quantum master equation

$$\begin{aligned} \dot{\rho} = & -i[H, \rho] + \sum_{\beta} \kappa_{\beta} (2\beta\rho\beta^{\dagger} - \beta\beta^{\dagger}\rho - \rho\beta\beta^{\dagger}) \\ & + \gamma_s (2\sigma_s^{-}\rho\sigma_s^{+} - \sigma_s^{+}\sigma_s^{-}\rho - \rho\sigma_s^{+}\sigma_s^{-}) \\ & + \gamma_a (2\sigma_a^{-}\rho\sigma_a^{+} - \sigma_a^{+}\sigma_a^{-}\rho - \rho\sigma_a^{+}\sigma_a^{-}), \end{aligned} \quad (6)$$

where ρ is the density matrix of the entire system, $\beta \in \{a, b, d\}$, κ_a , κ_b and κ_d are the decay rates of the optical cavity, microwave cavity, and the mechanical oscillator, respectively; γ_a and γ_s are the lifetime of the atomic and superconducting qubits, respectively. We characterize the transfer process for the given initial state by the conditional fidelity of the quantum state defined by $F_2 = \langle\psi_a|\rho|\psi_a\rangle$. By choosing the typical parameters: $\kappa_a = 1$ MHz, $\kappa_b = 1$ MHz, $\kappa_d = 0.1$ MHz, $\gamma_a = 0.1$ MHz and $\gamma_s = 0.1$ MHz, we plot, in Fig. 2, the fidelity F_2 as a function of the dimensionless time πgt , where we have obtained high fidelity $F_2 > 95\%$ of the state transfer. In particular, even with considerable mismatch (about $\pm 10\%$) of the coupling strength, $F_2 > 94\%$ can still be obtained. Therefore, the state transfer process is very robust to mismatch of the coupling strength to the ideal condition. In the above estimation, we have neglected the effect from the atoms in the optical lattice due to the following two reasons. Firstly, the cavity-assisted interaction can be effectively switched off. Secondly, if there is a small probability that it has not been switched, then the influence is that they will cause energy shift of the cavity mode. For N atoms, this shift is $Ng^2/\delta \sim Ng/10$ for $\delta \sim 10g$. For the ancillary atom, we may choose $\delta' = 10\Omega_A = 100g$; in this way $\lambda = 0.1g$ is still large enough for our manipulation purpose. With the above parameters, for $N = 5$, we obtain that the atom-induced energy shift is 0.5% of δ' , and thus can be safely neglected. Therefore, combining the process of storing the auxiliary atomic state into the TQM, we can obtain a fidelity $F = F_1 \times F_2 \approx 86\%$ of storing a superconducting qubit into the TQM.

In summary, we have proposed a hybrid system consists of a TQM with cold atoms and a superconducting qubit, which combines the advantages of both the noise resistance of topological qubits and the scalability of superconducting qubits. In particular, by introducing a quantum optomechanical setup, we have demonstrated that the superconducting qubit state can be efficiently transferred into the TSQ.

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[1] J. Q. You and F. Nori, Nature (London) **474**, 589 (2011); J. Clarke and F.K. Wilhelm, Nature (London) **453**, 1031 (2008).

[2] A. Kitaev, Ann. Phys. (N.Y.) **303**, 2 (2003); **321**, 2 (2006).

- [3] X.-L. Qi and S.-C. Zhang, *Rev. Mod. Phys.* **83**, 1057 (2011).
- [4] M. Z. Hasan and C. L. Kane, *Rev. Mod. Phys.* **82**, 3045 (2010).
- [5] C. L. Kane and E. J. Mele, *Phys. Rev. Lett.* **95**, 146802 (2005).
- [6] B. A. Bernevig, T. L. Hughes and S.-C. Zhang, *Science*, **314**, 1757 (2006).
- [7] L. Fu, C. L. Kane, and E. J. Mele, *Phys. Rev. Lett.* **98**, 106803 (2007).
- [8] J. E. Moore and L. Balents, *Phys. Rev. B* **75**, 121306 (R) (2007).
- [9] R. Roy, *Phys. Rev. B* **79**, 195322 (2009).
- [10] J. Ruseckas, G. Juzeliūnas, P. Öhberg, and M. Fleischhauer, *Phys. Rev. Lett.* **95**, 010404 (2005); S.-L. Zhu, H. Fu, C.-J. Wu, S.-C. Zhang, and L.-M. Duan, *ibid.* **97**, 240401 (2006); S.-L. Zhu, L.-B. Shao, Z. D. Wang, and L.-M. Duan, *ibid.* **106**, 100404 (2011).
- [11] Y.-J. Lin, K. Jiménez-García, and I. B. Spielman, *Nature* **471**, 83 (2011).
- [12] M. Chapman and C. Sá de Melo, *Nature* **471**, 41 (2011).
- [13] P. Wang, Z.-Q. Yu, Z. Fu, J. Miao, L. Huang, S. Chai, H. Zhai, and J. Zhang, *Phys. Rev. Lett.* **109**, 095301 (2012).
- [14] L. W. Cheuk, A. T. Sommer, Z. Hadzibabic, T. Yefsah, W. S. Bakr, and M. W. Zwierlein, *Phys. Rev. Lett.* **109**, 095302 (2012).
- [15] J.-Y. Zhang, S.-C. Ji, Z. Chen, L. Zhang, Z.-D. Du, B. Yan, G.-S. Pan, B. Zhao, Y. Deng, H. Zhai, S. Chen, and J.-W. Pan, *Phys. Rev. Lett.* **109**, 115301 (2002).
- [16] X.-J. Liu, Z.-X. Liu, and M. Cheng, *arXiv*: 1209.2990 (2012).
- [17] I. Bloch, J. Dalibard, and W. Zwerger, *Rev. Mod. Phys.* **80**, 885 (2008).
- [18] L. Jiang, G. K. Brennen, A. V. Gorshkov, K. Hammerer, M. Hafezi, E. Demler, M. D. Lukin, and P. Zoller, *Nature Phys.* **4**, 482 (2008).
- [19] T. J. Kippenberg and K. J. Vahala, *Science* **321**, 1172 (2008).
- [20] E. Verhagen, S. Deléglise, S. Weis, A. Schliesser, and T. J. Kippenberg, *Nature (London)* **482**, 63 (2012).
- [21] J. D. Teufel, D. Li, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, and R.W. Simmonds, *Nature (London)* **471**, 204 (2011).
- [22] R. J. Schoelkopf and S. M. Girvin, *Nature (London)* **451**, 664 (2008).
- [23] A. Y. Kitaev, *Physics-Uspekhi* **44**, 131 (2001).
- [24] J. Cho, *Phys. Rev. Lett.* **99**, 020502 (2007); A. V. Gorshkov, L. Jiang, M. Greiner, P. Zoller, and M. D. Lukin, *Phys. Rev. Lett.* **100**, 093005 (2008).
- [25] C. J. Hood, T. W. Lynn, A. C. Doherty, A. S. Parkins, and H. J. Kimble, *Science* **287**, 1447 (2000).
- [26] K. R. Brown, A.W. Harrow, and I. L. Chuang, *Phys. Rev. A* **70**, 052318 (2004).
- [27] F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, *Phys. Rev. Lett.* **99**, 093902 (2007); I. Wilson-Rae, N. Nooshi, W. Zwerger, and T. J. Kippenberg, *Phys. Rev. Lett.* **99**, 093901 (2007).
- [28] Y.-D. Wang and A. A. Clerk, *Phys. Rev. Lett.* **108**, 153603 (2012); L. Tian, *Phys. Rev. Lett.* **108**, 153604 (2012).
- [29] Z.-Q. Yin and F.-L. Li, *Phys. Rev. A* **75**, 012324 (2007).